Cycle counting using rainflow algorithm for fatigue analysis

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ABSTRACT

Rainflow algorithm is one of the widely accepted techniques of cycle counting in fatigue analysis for damage assessment in engineering components and structures. In this paper a brief summary of Rainflow cycle counting technique has been outlined. To demonstrate the efficacy of the developed Rainflow algorithm in a computer is being validated by numerous standard set of test problems/ simple sequences are described. An attempt has been made to illustrate the algorithm accuracy by demonstrating a number of simple examples. The examples that are included are (i) Available literature sequences, (ii) Decreasing amplitude sequence (iii) Increasing amplitude sequence and (iv) Constant amplitude sequence. Finally the results from analyzing a service load spectrum of a fighter aircraft is provided in this paper.

Keywords: Rainflow, Range pair-range, Cycle counting, Fatigue analysis

1. INTRODUCTION

In 1970's National Aerospace Laboratory, NLR, the Netherlands developed a two-dimensional counting procedure which was later baptized Range pair-range method[1]. At the same time and independently in Japan a method was developed which became known as Rainflow method. Both methods yield exactly the same result, i.e., they extract the same range pairs and single ranges from the sequence, thus combining the range pair counting principle and the single range counting principle in one counting method. However, the descriptions of the methods are very different.

The different cycle counting techniques available[2] are: (i) Peak count, (ii) Mean crossing count (iii) Peak-Trough count, (iv) Level crossing count, (v) Fatigue meter count, (vi) Range count, (vii) Range mean count, (viii) Range pair count, (ix) Markov matrix count, and (x) Rainflow cycle counting. The other types of cycle counting methods discussed by Yung-Li Lee[3] are: 1. One-parameter cycle counting methods, (i) Level crossing cycle counting, (ii) Peak-Valley cycle counting, (iii) Range counting, 2. Two-parameter cycle counting method:- (i) Rainflow cycle counting method can faithfully represent variable-amplitude cyclic loading. It can identify events in a complex loading sequence that is compatible with constant-amplitude fatigue data. 3. Three-point cycle counting method and 4. Four-point cycle counting method.

Nothing is as important to the success of an algorithm as the accuracy of its performance. Indeed, it is a single pass (one go) analysis (no iterations), the counting of the number of cycles and the range of each of these cycles are have to be exact. The loads occurring in actual service in aircrafts/ helicopters are challenging tasks to predict with high accuracy. As a part of certification process, the measured data will have to be analyzed and in some way quantified.

2. RAINFLOW CYCLE COUNTING PROCEDURE

Generally, damage due to cycles is quantified by considering Wöhler-curves[4] from constant amplitude tests and by employing an approach that covers the influence of mean stresses. Rainflow cycle counting is a procedure for determining damaging events in variable amplitude loading. Closed hysteresis loops are registered in the stress-strain path. These loops are treated as one cycle of the corresponding constant amplitude loading.

In fatigue, damage is usually interpreted as crack growth. It is well known that fatigue crack growth depends more on the effective stress or strain range than on the applied ranges. The effective values result from the crack closure phenomenon due to which the applied stress range is reduced; such a mechanism strongly depends on the load history.

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2.1 Preliminary treatment of sequence

Application of the rainflow method and storage of its results requires a preliminary treatment of the loading, which consists of sampling, extraction of extrema, and finally quantification of the values into classes.

A service measurement generally consists of monitoring of a continuous variable over time (an analogue signal). Treatment of the sequence by computer requires discrete digital values, sampled in a way that adequately represents the real evolution of the variable. The rainflow counting method needs only the successive extrema of the loading sequence, which have to be extracted from the sampled sequence.

To store the results, and for speed of analysis, it may be necessary to quantify the possible values. In practice, the total range of the sequence is divided into k classes of constant-width interval or steps, and all the values (peaks or valleys) located in a given class are replaced by a representative value of this class (the mean value is usually chosen). Standard practice in industry is to take the number of classes equal to 64[5].

2.2 Rainflow Counting Method

The rain-flow algorithm has been adapted since 1970s for use by computers. The first algorithm of rainflow counts, by Downing[6], a history of peaks and valleys in sequence, which has been rearranged to begin and end with the maximum peak (or valley). Due to the simple reason that it requires rearrangement of load history this algorithm is completely ignored, in this article.

The second algorithm, by Downing, Glinka[6,7]counts a history of peaks and valleys in sequence as they occur. It calculates the same ranges and means as first algorithm, which required that the history be rearranged to begin and end with the maximum peak (or minimum valley).The algorithm has been implemented in an in-house, program that caters for long stress histories too.

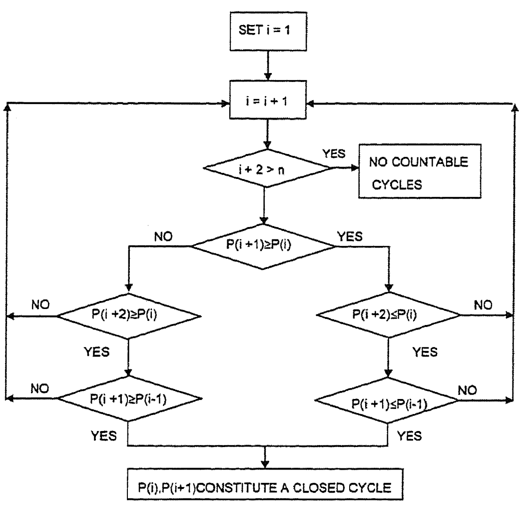

Figure 1. Flow chart for counting first closed cycle in n peak-troughs

The following are the rules[8] for Rainflow counting:

i) Rain will flow down the roof initiating at the inside of each peak. When it reaches the edge it will drip down.

ii) The rain is considered stopped when it reaches another flow from top.

iii) The rain stops when it comes opposite a maximum more positive than the maximum where it started (for left to right flow).

iv) The rain stops when it comes opposite a minimum more negative than the minimum from which it started (for right to left flow).

The rainflow technique as a mathematical algorithm is illustrated in Fig. 1.

The rainflow counting method (RCM) has been designed to count cycles (consisting of a pair of reversals) or half-cycles (reversals) in accordance with the stress-strain response of the material. For the history in Fig. 2, a_1-b_1-a_1 should be a cycle that can be counted by the RCM. For the history in Fig.3, the cycle a_1-b_2-a_1 should be a cycle that cannot be counted by RCM.

2.3 Modified Rainflow Counting Method

Modified rainflow counting method (MRCM)[9] is represented as the strain/ stress-time history is plotted so that the time axis is vertically downward (Fig. 4), and the lines connecting the stress peaks and valleys are imagined to be a series of pagoda roofs. Several rules are imposed on rain dripping down these roofs so that cycles and half-cycles can be counted as explained in previous section. Rain flow begins successively at the inside of each stress peak or valley, and every part of the history is counted once and only once. The in-house developed program is governed by the rules and do recognizes all these examples (Fig. 2 and Fig. 3) so it is regarded as MRCM.

3. EXAMPLES

3.1 Example No. 1 Modified Rainflow Counting, Anthes[10]

The strain-time history considered is shown (Fig. 4) leading to the stress-strain path (Fig. 5). Table 1a is the input to the computer program consisting of sequence of data points. Table 1b is the corresponding output of computer program consisting of number of cycles and relevant information.

Table 1a. Example sequence (Fig. 4)

POINT	1	2	3	4	5	6	7
Strain	1.1	-4.6	-0.2	-3.5	-0.8	-2.7	6.0

Table 1b. Program output for Table 1a.

RANGE	MEAN	CYCLE NO.	PEAK	VALLEY	PEAK
0.1900E+01	-0.1750E+01	1	0.6000E+01	-.2700E+01	-.8000E+01
0.3300E+01	-0.1850E+01	2	0.6000E+01	-.3500E+01	-.2000E+00
0.1100E+01	0.5500E+00	3	-.4600E+01	0.1100E+01	0.0000E+00
0.1060E+02	0.7000E+00	4	-.4600E+01	0.6000E+01	-.4600E+01

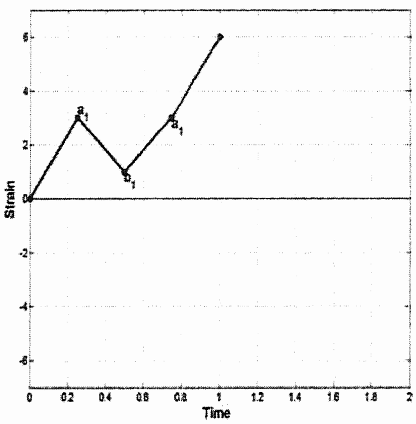

Figure 2. Positive amplitude sequence

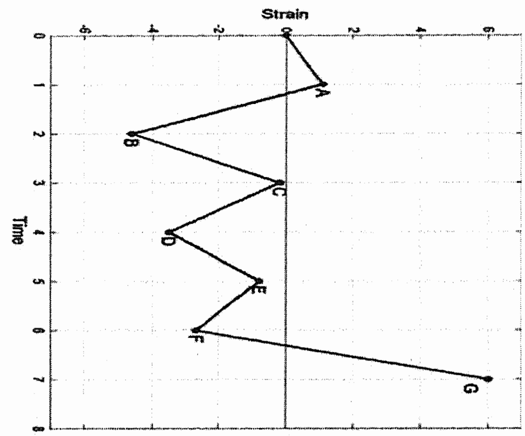

Figure 4. Srain-time history sequence

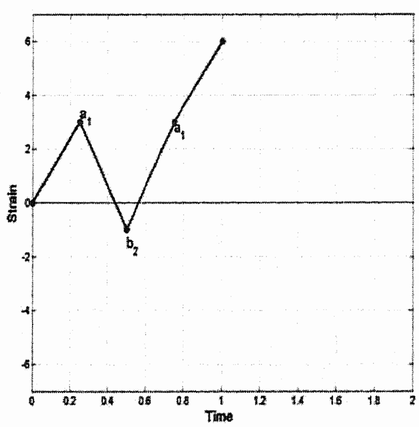

Figure 3. Positive negative sequence

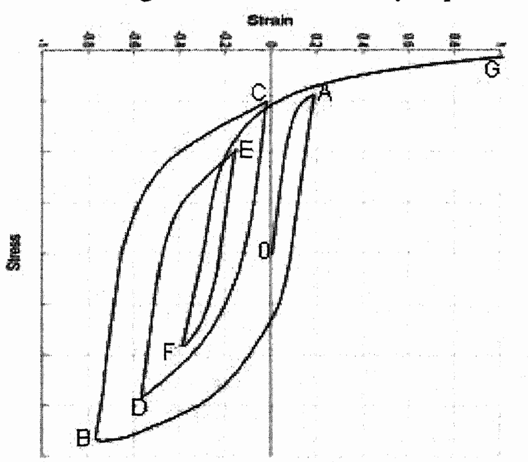

Figure 5. Example of the rainflow cycle counting

3.2 Example No. 2 A sequence from: ASTM E1049

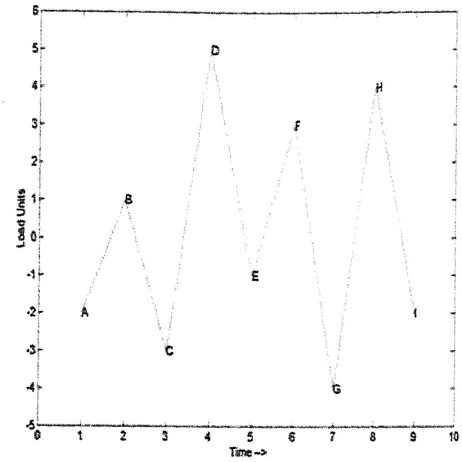

Figure 6. Example sequence plot of load versus time

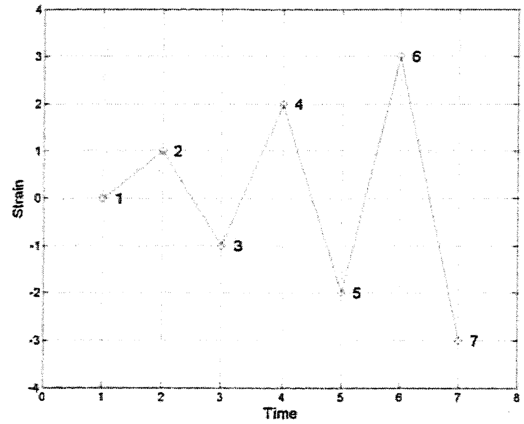

Figure 8. Plot of increasing magnitude sequence

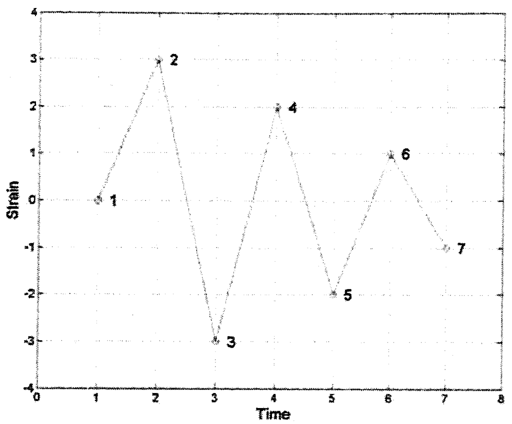

Figure 7. Plot of decreasing magnitude sequence

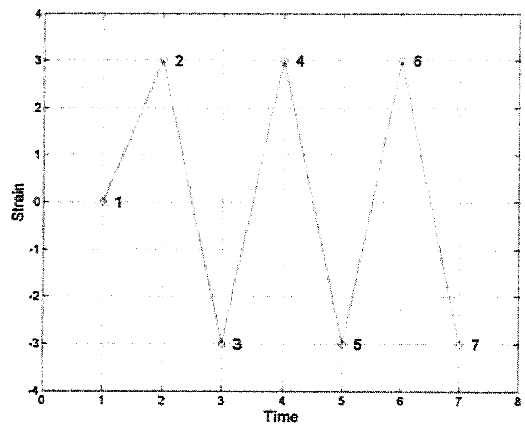

Figure 9. Plot of constant magnitude of load sequence

Table 2. List of number of cycles counted from the reference[2] ASTM E1049.

RANGE (UNITS)	CYCLE COUNTS	EVENTS
10	0	
9	0.5	D-G
8	1.0	C-D, G-H
7	0	
6	0.5	H-I
5	0	
4	1.5	B-C, E-F
3	0.5	A-B
2	0	
1	0	

RANGE	MEAN	CYCLE NO.	VALLEY	PEAK
4.0	-1.0	1	-3.0	1.0
4.0	1.0	2	3.0	-1.0
6.0	1.0	3	-2.0	4.0
9.0	0.5	4	-4.0	5.0

Table 3. Program output for table 2 sequence

In this problem the range parameter can be easily identified. The two half cycles AB and BC form one cycle of range 4 (Table 2). The EF form one complete cycle of range 4. Another two half cycles GH and HI forms one cycle of range 6. The remaining two half cycles CD and DG forms one cycle of range 9. Thus, these problems exactly validate the algorithm implemented.

The table 3 gives the list of number of cycles counted for the example sequence 2 shown in figure 6 from the program developed. The difference between the valley (trough) and peak is the **Range**, and the average of valley and peak is the **Mean**.

3.3 Example No. 3 Decreasing amplitude sequence

A history of decreasing (converging) sequence as in Fig. 7.

Table 4a. Input sequence

POINT	1	2	3	4	5	6	7
Strain	0	3	-3	2	-2	1	-1

Table 4b. Output cycles

RANGE	MEAN	CYCLE	PEAK	VALLEY	PEAK
2.0	0.0	1	3.0	-1.0	1.0
4.0	0.0	2	3.0	-2.0	2.0
6.0	0.0	3	3.0	-3.0	3.0

The rainflow cycle counting method is based on the analogy of raindrops falling on a pagoda roof and running down the edges of the roof. The history is rotated so that the time axis is vertical and visualizing water poured on top. The flow starts at the top (beginning) of the history and is also initiated at the inside of each maximum or minimum. Flow is stopped when it strikes flow descending from above, or a point opposite a maximum or minimum whose magnitude exceeds that at a point from which it is started. Flow also stops at the end of the record. Each flow is a half cycle, and there is a complementary half cycle of opposite sign elsewhere in the complete record. In this method, it cannot be assumed that a converging history would simply skip the other lower peaks and no cycles are counted.

This example (the decreasing sequence table 4a), demonstrate that, the rainflow cycle counting algorithm render correct number of cycles, as in table 4b. Similar statements hold good for diverging history too (table 5a). In fact, in both cases exact number of cycles can be identified (table 5b) due to the fact that input data is reversed and maintain the magnitude and number of data points are equal.

3.4 Example No. 4 Increasing amplitude sequence

A history of increasing (diverging) sequence as in Fig. 8.

Table 5a. Input sequence

POINT	1	2	3	4	5	6	7
Strain	0	1	-1	2	-2	3	-3

Table 5b. Output cycles

RANGE	MEAN	CYCLE	PEAK	VALLEY	PEAK
4.0	0.0	1	3.0	-2.0	2.0
2.0	0.0	2	3.0	-1.0	1.0
6.0	0.0	3	3.0	-3.0	3.0

3.5 Example No. 5Constant amplitude sequence

Constant amplitude loading history sequence, as in Fig. 9.

Table 6a. Input sequence

POINT	1	2	3	4	5	6	7
Strain	0	3	-3	3	-3	3	-3

Table 6b. Output cycles

RANGE	MEAN	CYCLE	PEAK	VALLEY	PEAK
6.0	0.0	1	3.0	-3.0	3.0
6.0	0.0	2	3.0	-3.0	3.0
6.0	0.0	3	3.0	-3.0	3.0

For constant amplitude loading cycle (Table 6a) the range is constant and the mean is zero, the same are predicted as in the table 6b. Again for the seven data points that are considered number of cycles obtained is three. This means that there is no residue or surplus data points left out.

The cases of example numbers 3 and 4 are discussed as the divergent-convergent history by Nie Hong[9], as a single loading history. However, it is interesting to see in both cases of converging and diverging sequence; obtained number of cycles is three.

3.6 Example No. 6 Flight Data Record (FDR) sequence

An actual in-flight recorded data of a fighter aircraft is obtained from a flight of approximately 2826 second (47.6 minutes), of one complete flight duration is plotted in Fig. 10. This plot shows the data after it has been filtered for successive peaks/ troughs and very small amplitudes. Resulting in peak and through points, are as follows:

1.2062 1.5051 1.2062 2.4016 0.9074 2.1027 1.2062 1.8637 0.7879 1.9234 0.8477 4.4934 1.8637
6.0473 1.2062 3.5371 1.8039 2.5809 1.8637 4.2543 2.5211 3.3578 1.3856 5.6887 2.0430 3.1188
1.5648 2.7602 0.9074 1.2062 0.6684

The corresponding output of cycle counts (CY) from the program is in table 7.

Figure 10.Plot of g-vertical (y-axis) or load factor versus time.

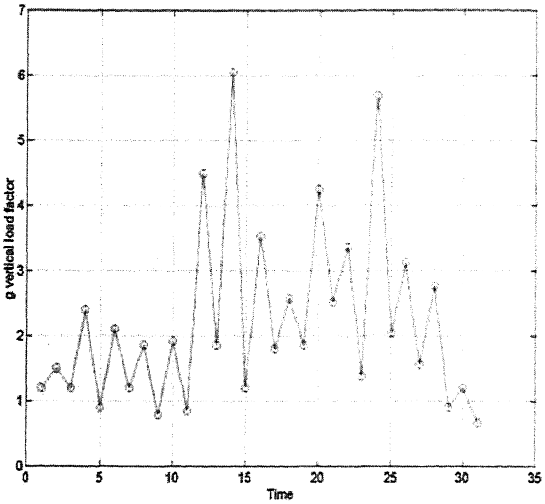

Table 7. Program output for g vertical sequence

RANGE	MEAN	CY	PEAK	VALLEY	PEAK
0.8367	2.9395	1	1.3856	3.3578	2.5211
1.0758	2.5809	2	1.5648	3.1188	2.0430
1.1954	2.1625	3	0.9074	2.7602	1.5648
0.2988	1.0568	4	0.6684	1.2062	0.9074
0.2989	1.3556	5	2.4016	1.2062	1.5051
0.6575	1.5350	6	0.7879	1.8637	1.2062
1.1953	1.5050	7	0.7879	2.1027	0.9074
1.0757	1.3856	8	4.4934	0.8477	1.9234
1.6137	1.5947	9	4.4934	0.7879	2.4016
2.6297	3.1786	10	6.0473	1.8637	4.4934
5.0203	3.1786	11	6.0473	0.6684	5.6887
0.7172	2.2223	12	4.2543	1.8637	2.5809
1.7332	2.6705	13	4.2543	1.8039	3.5371
2.8687	2.8200	14	6.0473	1.3856	4.2543
4.8411	3.6267	15	6.0473	1.2062	6.0473

4. CONCLUDING REMARKS

The studies show that the rainflow method emerges as a reference method that can be used to store service measurements in a form suitable for fatigue analysis of materials and structures from the data in order to simulate the sequence. It presents a decisive advantage compared with the spectral or statistical analysis of the sequence because it makes it possible both to analyze and to represent variable amplitude loading. It is possible to deduce, from the rainflow counting results, the results of other counting methods (peak and valleys, level crossing, range pairs). In all these methods the reverse is not true. About 50 different sequences were computed number of cycles and it was verified that there were no residue data points. Also their counted number of cycles matches with those given in the reference data sources.

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